Griffiths phases in the strongly disordered Kondo necklace model

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The effect of strong disorder on the one-dimensional Kondo necklace model is studied using a perturbative real-space renormalization group approach which becomes asymptotically exact in the low energy limit. The phase diagram of the model presents a random quantum critical point separating two phases; the *random singlet phase* of a quantum disordered XY chain and the random Kondo phase. We also consider an anisotropic version of the model and show that it maps on the strongly disordered transverse Ising model. The present results provide a rigorous microscopic basis for non-Fermi liquid behavior in disordered heavy fermions due to Griffiths phases.

An understanding of the effects of randomness on the quantum critical point (QCP) of the $d = 1$ Kondo necklace (KN) model [1] is relevant for the study of disordered heavy fermions systems with non-Fermi liquid behavior [2, 3, 4]. Recently a *non-perturbative* real space renormalization group (RG) was presented showing that *weak disorder is an irrelevant perturbation* near the QCP of a $d = 1$, anisotropic, pure KN model [5]. This result is in agreement with the generalized Harris criterion [6, 7] for irrelevance of disorder, $\nu > 2/d$, where $\nu = 2.24$ is the value obtained for the correlation length exponent of the QCP of the pure anisotropic system [5]. On the other hand different approaches have been proposed to describe the non-Fermi liquid behavior of disordered heavy fermions that rely on the *relevance* of disorder [2, 3]. In order to settle this important point we investigate here the one-dimensional KN model in the case of *strong disorder* using a generalization of a perturbative real space RG approach. This is the Ma-Dasgupta-Hu method [8] which has been extended by Fisher [9, 10] and others [11] mostly for the study of random quantum spin chains and the random transverse-field Ising model $(RTIM)$. In this Letter we obtain an exact mapping of the strongly disordered KN model into a problem of quantum spin chains. For the anisotropic KN model, the mapping is in the random transverse-field Ising model $(RTIM)$. The general mapping allows to apply directly to the KN problem the results of extensive work done recently on random quantum spin chains [9, 10, 11, 12, 13, 14, 15]. This includes the existence of Griffiths phases with the characteristic singular behavior of different thermodynamic quantities at low temperatures. The present work provides a rigorous microscopic justification for the non-Fermi liquid behavior of disordered heavy fermions due to the existence of a Griffiths phase in the neighborhood of a random QCP [3].

The one-dimensional KN model is defined by the Hamiltonian,

$$H = \sum_{i=1}^{L-1} W_i(\sigma^x{}_i\sigma^x{}_{i+1} + \sigma^y{}_i\sigma^y{}_{i+1}) + \sum_{i=1}^{L-1} J_i\vec{S}_i.\vec{\sigma}_i, \quad (1)$$

where σ^μ and S^μ, $\mu = x, y, z$ are spin-1/2 Pauli matrices denoting the conduction electrons and the spins of the local moments, respectively. The sites i and $i + 1$ are nearest-neighbors on a chain of L sites. The local Kondo interactions, $J_i > 0$ and the hopping energies $W_i > 0$ are uncorrelated quenched random variables with probability distributions, $P_J(J_i)$ and $P_W(W_i)$. In the anisotropic version of the model, $(X - KN)$, the band of conduction electrons is represented just by an Ising term, $\sum_{i=1}^{L-1} W_i\sigma^x{}_i\sigma^x{}_{i+1}$. The full isotropic KN model in Eq. (1) will be refered from now on as the $XY - KN$ to avoid confusion.

The KN model was proposed by Doniach [1] to study heavy fermions and emphasizes magnetic degrees of freedom neglecting charge fluctuations. It incorporates the essential physics of these systems which results from the competition between Kondo effect and magnetic ordering. In the absence of disorder, the models above have distinct behavior. For the $X - KN$ there is an unstable fixed point at a finite value of (J/W) separating an antiferromagnetic phase from a spin compensated, Kondo-like phase [5]. For the $XY - KN$, any interaction $J > 0$ gives rise to a dense Kondo state [5, 16, 17, 18, 19].

In this Letter, in order to implement the perturbative RG method for the KN models, we consider the conduction electrons σ_i and the local moments S_i arranged in a chain as shown in Fig.1. Next we choose the largest interaction in the chain,

$$\Omega = \max\{W_i, J_i\} \quad (2)$$

If the strongest interaction is a Kondo coupling (*bond*) between a local moment and a conduction electron, for
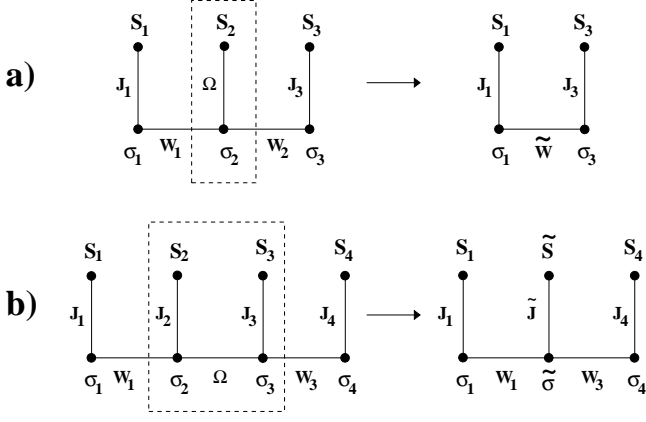
FIG. 1: The decimation processes in the case the strongest interaction Ω is either a) a bond or b) a hopping .

example, $\Omega_I = J_2$, the local moment S_2 and the conduction electron σ_2 are decimated out yielding an effective hopping $\tilde{W}$ between the conduction electrons σ_1 and σ_3 at neighboring sites which is obtained in second order perturbation theory (see Fig. 1).

If the strongest interaction is a hopping term, say, $\Omega_I = W_2$, the four spins σ_2, S_2, σ_3 and S_3 are considered as a *cluster*. This is replaced by an effective two-spin cluster consisting of renormalized, local moment $\tilde{S}$ and conduction electron $\tilde{\sigma}$ coupled by a new effective local Kondo interaction, $\tilde{J}$ (see Fig. 1). Thus, after decimating a strong interaction, W_i or J_i, we have an effective Hamiltonian with two less spin degrees of freedom and all couplings $< \Omega_I$.

The RG transformation gives, in the case the strongest interaction is a bond, an effective hopping,

$$\tilde{W} = \frac{W_1 \, W_2}{\kappa \Omega} \qquad (3)$$

and in the case it is a hopping we obtain,

$$\tilde{J} = \frac{J_2 \, J_3}{\kappa \Omega} \qquad (4)$$

The new Hamiltonian has exactly the same form as the original one, but now the system is formed by spin clusters and effective bonds. Note that the resulting flow equations, Eqs. (3) and (4), present a duality between W and J. We find that for the $X - KN$ model the parameter $\kappa = 1$, so that the recursion relations for this model map exactly into those of the $RTIM$ [10]. For the $XY - KN$ model, we get $\kappa = 4/\sqrt{6} \approx 1.63$.

The method is implemented numerically on samples of sizes up to $L = 2^{18}$ and averages over 10^2 configurations. We use rectangular distributions for the local bonds and hopping terms. Periodic boundary conditions are applied. The relevant parameter is the ratio (J_0/W_0) of the cut-offs of the original distributions. Furthermore we take $W_0 = 1$ such that J_0 is taken as the control

parameter. The dual nature of the recursion relations allows to locate the random QCP at $J_0 = 1$ for any κ.

We measure the distance to the random QCP by the variable

$$\delta = \frac{< \ln J > - < \ln W >}{var(\ln J) + var(\ln W)} \qquad (5)$$

where $< - >$ means average over quenched disorder and $var(x)$ denotes the variance. Of course $\delta = 0$ for $J_0 = 1$.

At the QCP, $J_0 = 1$ or $\delta = 0$, the parameter $\kappa \geq 1$ is irrelevant [20] and we obtain for both models a behavior associated with a *random singlet phase* of the local moments. This is characterized by the fact that the fixed point distributions of bonds, $P_J(J_i)$, and hoppings, $P_W(W_i)$, have power-law forms,

$$P_J(J_i, \Omega) = \frac{\alpha_J}{\Omega} \left(\frac{\Omega}{J} \right)^{1 - \alpha_J} \theta(\Omega - J)$$

$$P_W(W_i, \Omega) = \frac{\alpha_W}{\Omega} \left(\frac{\Omega}{W} \right)^{1 - \alpha_W} \theta(\Omega - W) \qquad (6)$$

with the exponents α_W and α_J depending on the cut-off Ω. They are given by,

$$\alpha_W = \alpha_J = -\frac{1}{\ln \Omega} \qquad (7)$$

The physical behavior of the thermodynamic quantities as a function of temperature T, which arises from such distributions of interactions is extensively described in the literature [10, 11, 21]. It is given by power laws with logarithmic corrections.

For the anisotropic KN model, the QCP at $J_0 = 1$ separates a disordered antiferromagnetic phase $(J_0 < 1)$ from a dense Kondo compensated phase $(J_0 > 1)$. The phase for $J_0 < 1$ of the $XY - KN$ model, where $P_J(J_i)$ becomes negligeable under iteration, is identified, from the limit $J_0 = 0$, as a random singlet phase. In fact this limit corresponds to the strongly disordered XY quantum chain which, as shown by Fisher [9], exhibits a random singlet phase. The nature of the disordered Kondo compensated phase for $J_0 > 1$ is very similar in both KN models and will be discussed in detail below.

Above and below, but close to criticality, the recursion relations in Eqs. (3) and (4), give rise to Griffiths phases with a range $0 < |\delta| < \delta_G$. In each side of the QCP such phases are dominated by rare, very large clusters of the opposite phase. They are also characterized by power law behavior of the probabilities distributions, as in Eq. (6), but in this case with exponents which depend on the distance δ to the QCP. As an illustration, we show in Fig. 2 the exponent α_J, in the disordered phase $(J_0 > 1)$ as a function of the cut-off Ω, as Ω is reduced under iteration, for different distances $(\delta < \delta_G)$ of the random QCP. For small δ, sufficiently close to the QCP, the finite intercept for $(-1/\ln \Omega) \to 0$ is directly proportional to this distance δ [10].
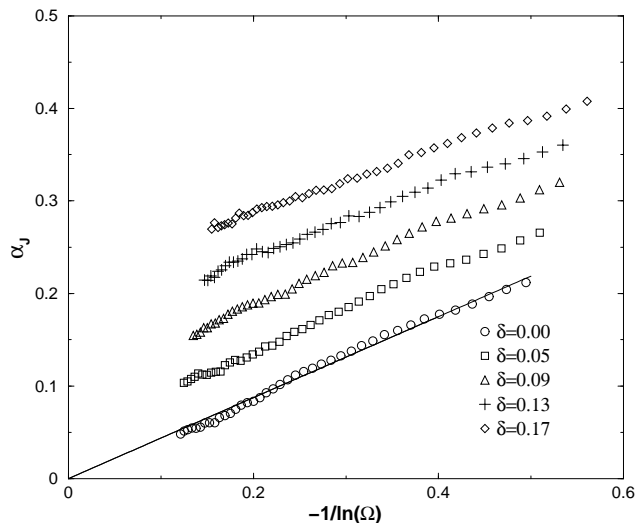
FIG. 2: The exponents α_J of Eq. (6) at the disordered dense Kondo phase ($J_0 > 1$) of the $X - KN$ model for different distances δ to the random QCP in the case of rectangular gapless distributions. The abscissa is the inverse of the logarithm of the effective cut-off.
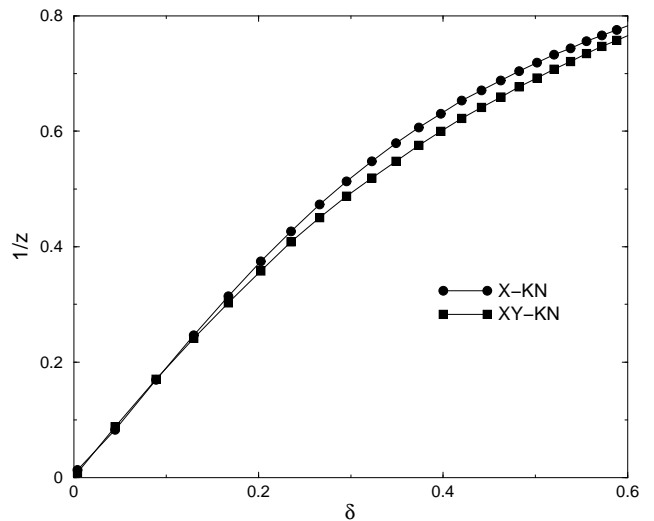

FIG. 3: The inverse of the dynamic exponents z for the $X - KN$ and $XY - KN$ models as function of the distances to the random QCP ($\delta > 0$). They are obtained iterating numerically the recursion relations starting from gapless rectangular distributions. The dynamic exponents determine the singularities of the thermodynamic quantities with decreasing temperature in the Griffiths phases (see text).

The range δ_G of the Griffiths phases in control parameter space depends on the original distributions of the interactions. For gapless distributions, i.e., those unbound from below, the Griffiths phases extend all over the phase diagram [12, 13]. For bounded distributions from below, i.e., with a low energy gap, they have a finite extension around the QCP.

The divergence of the susceptibility in the Griffiths phase is given by, $\chi_0(T) \propto T^{1/z_\kappa(\delta)-1}$ where z is the *dynamical exponent* [12, 13]. The behavior of the specific heat in this phase is given by, $C_V(T)/T \propto T^{-1+1/z_\kappa(\delta)}$ and for small magnetic fields H, the magnetization, $M \propto H^{1/z_\kappa(\delta)}$. The z exponent has the usual meaning of a dynamic exponent in quantum phase transitions in that it relates length and time scales. It is an invariant of the RG equations, Eqs. (3, 4), and assumes the values $z = \infty$ and $z = 1$ at the random QCP ($\delta = 0$) and at the border of the Griffiths phase ($|\delta| = \delta_G$), respectively[13, 14].

Fig. 3 shows the dynamic exponents $z_\kappa(\delta)$ for both $X - KN$ and $XY - KN$ models in the Griffiths phase, at $J_0 > 1$, for rectangular unbound distributions. Due to its invariance along the iteration process this exponent is very useful to characterize the temperature dependence of different thermodynamic quantities in all the Griffiths phase. For the original gapless distributions used to obtain $z_\kappa(\delta)$ in this figure, this phase extends for all $\delta > 0$ [13, 14]. Note from Fig. 3 that for sufficiently small δ, $z_\kappa(\delta)^{-1} \propto \delta$ [9, 12].

Starting the iteration with a distribution $P_J(J_i)$ with a gap, in the disordered Kondo phase ($J_0 > 1$) we obtain for both the $X - KN$ and $XY - KN$, besides the Griffiths phase for $\delta < \delta_G$, a strongly disordered *random Kondo phase* (RKP) for $\delta > \delta_G$. This phase consists essentially of a collection of isolated Kondo singlets with a distribution of excitation energies. It is natural then to describe such phase using a distribution of Kondo temperatures [2].

In Fig. 4 we show the uniform susceptibility as a function of temperature, $\chi(T)$, for the $X - KN$ model, calculated far away from the random QCP, deep in the disordered Kondo phase for two initial distributions $P_J(J_i)$. When the original distribution of Kondo couplings J_i has a finite gap Δ_J at low energies, the susceptibility (solid line) does not diverge at low T as shown in the figure. In this case the weakly disordered Griffiths phase extends to $\delta < \delta_G = 0.358$ which corresponds to a gap in the original $P_J(J_i)$ of $\Delta_J < 0.156$. The susceptibility shown is for $\delta = 0.44 > \delta_G$, in the RKP. It is clear that in this case the fixed point distribution of gaps or Kondo temperatures does not become sufficiently singular in the RKP to yield a diverging susceptibility. In the same figure we show the temperature dependent susceptibility for an initial unbound distribution of interactions, i.e., a rectangular distribution with no gap. The susceptibility now diverges as $T \to 0$. Since in this case of gapless distributions, the Griffiths phase extends all over the disordered Kondo region, $\delta > 0$, the divergence of $\chi(T)$ is clearly associated with this Griffiths phase. Similar behavior is found for the $XY - KN$ model.

In summary, we have studied the effects of strong disorder in the KN model for heavy fermion systems using a perturbative RG approach. Differently from the case
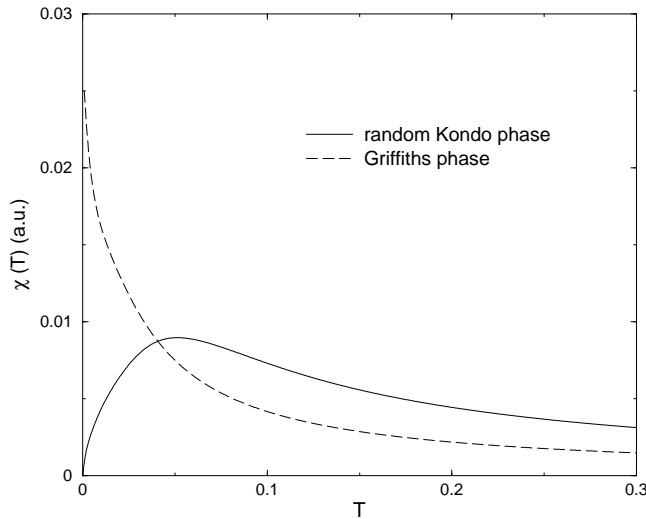
FIG. 4: The uniform susceptibility of the $X - KN$ model as a function of temperature for two initial distributions of bonds and the same distance $\delta = 0.44$ of the random QCP. For a gapped distribution (solid line), for which $\delta > \delta_G \approx 0.36$, and the system is in the RKP. For a gapless distribution (dashed line) such that the system is in the Griffiths phase. Temperature is in units of W_0.

of weak disorder [5], strong disorder is a relevant perturbation and gives rise to Griffiths phases which for distributions with a gap extend over finite regions of the phase diagram, above and below the QCP. For $J_0 > 1$, in the strongly disordered regime above the Griffiths phase, we find a random Kondo phase which consists of a collection of isolated singlets with random excitation energies. It is natural to describe such a phase by a model with a distribution of Kondo temperatures [2]. The susceptibility however does not diverge at low temperatures in the RKP. We observe a diverging susceptibility in the case of lower unbound distributions where the Griffiths phase extends all over the disordered region. Consequently our results indicate that a diverging $\chi(T)$ is due to a Griffiths phase. It can not be obtained in a model of isolated Kondo singlets with random Kondo temperatures. A fixed point distribution $P_J(J_i)$ of the power law type, that gives rise to a diverging $\chi(T)$, is incompatible with the existence of isolated singlets. In the Griffiths phases, the temperature dependence of the thermodynamic quantities is expressed in terms of the dynamic exponent z. We have obtained this exponent for both KN models as a function of the distance to the QCP for gapless distributions.

At the random QCP of both the $X - KN$ and the $XY - KN$ there are random singlet phases characterized by thermodynamic quantities with power law behavior, arising from distributions of interactions like in Eqs. (3) and (4), with weakly temperature dependent exponents. For the disordered $X - KN$ model there is a disordered antiferromagnetic phase for $J_0 < 1$ ($\delta < 0$).

The dense Kondo phase for $J_0 > 1$ ($\delta > 0$) is similar in both KN models studied. For the $XY - KN$ with $J_0 < 1$ ($\delta < 0$) there is a random singlet phase coexisting for some range of δ, which depends on the original distributions, with a weakly disordered Griffiths phase [10]. The exact mapping of the recursion relations for the $X - KN$ on those of the $RTIM$ allows to carry on the extensive results obtained on the latter to the present former model. It enables us to show unambiguously the existence of non-Fermi liquid behavior in strongly disordered heavy fermions associated with Griffiths phases.

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